



Potential models, sum rules, color singlet model versus NREFT in Heavy Quarkonium

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I briefly review how nonrelativistic effective field theories solve old puzzles and open problems in heavy quarkonium physics.

1 Historical recollection

Today quarkonium is a key issue in most of the accelerator experiments (see [1, 2]) where millions and in some cases billions of heavy quarkonium states are being produced. Historically quarkonium has been one of the most important playgrounds for our understanding of the strong interactions.

In perspective, the seventies were dominated by the J/ψ discovery, around 3095 MeV with a lifetime about 1000 times longer than that of other particles of comparable mass. This was known in the particle physics community as the November revolution. During the one year after the discovery more than seven hundreds papers were written related to J/ψ . Most of the papers dealt with quark and antiquark in a colour singlet state bound by a phenomenological potential. Since the structure of the energy levels of charmonium and the soonly after discovered bottomonium is somehow intermediate between a Coulomb and an harmonic oscillator structure and due to the asymptotic freedom idea [3], the basic form of the static potential is taken as a superposition of a Coulomb and a linear potential:

$$V_0 = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r \quad (1)$$

where α_s , the strong coupling constant at some scale, and σ , the string tension, have to be fit on the data. Such static singlet potential, together with similar phenomenological spin-dependent corrections, when used inside a Schrödinger equation gives an overall successful description of heavy quarkonium spectra and decays [4, 5, 2]. However, already in these pioneering papers the problem arised of the relation of these potential models to QCD and to the QCD parameters. Starting with the work of Wilson, an effort [6, 7] was devoted to relate the singlet potential, static and relativistic corrections, to QCD average values of Wilson loop and field strength insertions into the Wilson loop, which are objects easy to evaluate in lattice QCD [8]. However, it turned out that these results for the potentials in terms of Wilson loops were inconsistent with the one loop calculation of the potentials in perturbation theory [7].

The color singlet potential model encountered soon concrete problems also in the calculation of inclusive annihilation decays rates of heavy quarkonium states into light hadrons (hadronic) and photons and lepton pairs (electromagnetic). It was assumed that the decay rate of the quarkonium state factored into a short distance part f , calculated in perturbative QCD as the annihilation rate of the heavy quark and antiquark, and a long distance nonperturbative part given in terms of the quarkonium wave function (or its derivatives) evaluated at the origin:

$$\Gamma = f(\alpha_s(m)) \cdot |\psi(0)|^2. \quad (2)$$

Explicit calculations at next to leading order in α_s in perturbation theory for S and for P wave decays into photons supported the factorization assumption which could not, however, be proved on general grounds for higher orders of perturbation theory. Indeed, in the case of P wave decays into light hadrons, it turned out that at order α_s^3 the factorization was spoiled by logarithmic infrared divergences. The same problem appeared in relativistic corrections to the annihilation decays of S wave states [9]. Therefore, potential models were unable to supply at higher order an infrared finite prediction for the inclusive decays.

The eighties witnessed the success of the sum rules. It was a distinctive prediction of the sum rules that as a result of QCD the η_c mass has to be located around 3 GeV and not at the much smaller value of 2.83 GeV claimed at that time by the experiments [10]. The later discovery of the genuine η_c state with mass 2.98 GeV at Stanford was a great success of QCD and gave motivations for further work on the sum rules. Sum rules work in terms of Wilson operator expansion. The Green function of the quark-antiquark pair injected in the vacuum is expanded in terms of perturbative coefficients and local nonperturbative objects like the gluon condensate and the quark condensate. Physically as far as the $Q\bar{Q}$ distance r is held smaller than the confinement scale $\Lambda_{\text{QCD}}^{-1}$ the binding between the quark is perturbative and the propagation of the heavy pair is taking place in nonperturbative “external” vacuum fields. In such situation the interaction with the nonperturbative (at the scale Λ_{QCD}) vacuum gluonic fields can be expanded

in multipoles and the leading contribution of the vacuum fields to the quarkonium energy levels is proportional to the gluon condensate $\langle 0|F_{\mu\nu}^a(0)F_{\mu\nu}^a(0)|0\rangle$ [11]. Such condensate corrections are incompatible with the genuine potential described in the previous paragraph. Precisely, the effects of the nonperturbative fluctuations of the gluonic field cannot be expressed in terms of a singlet $Q\bar{Q}$ interaction potential and one should consider states that contain both the $Q\bar{Q}$ pair and a gluonic excitation, including the case in which both of them are in a color octet [11]. In brief, sum rules results for heavy quarkonia seemed to be in strong contradiction with the potential model picture.

The nineties were dominated by the data on prompt production of charmonium at Fermilab. The first hadron collider measurements of inclusive charmonium production at CERN and from the CDF collaboration at Fermilab could not separate charmonium produced in hard scattering reactions from charmonium produced in weak decays of B mesons. Thus the comparison with theory was uncertain. A rigorous test of the colour singlet production model (i.e. the assumption that the two quark were produced in a color singlet state) became possible with the CDF data on direct charmonium production where the contributions from the B -decays had been removed using microvertex-detection. With these data it became clear that the colour-singlet model failed dramatically when confronted with the experimental results[12].

This historical excursion ends with several puzzles and open problems related to the existence or nonexistence of a quark antiquark singlet potential, and the validity or not of a color singlet picture. Today the stage is taken by the many experiments accumulating high statistic data samples at quarkonium resonances and with several production mechanisms. It becomes therefore even more relevant to clarify the open theoretical problems and to supply a clean and under control QCD picture of these systems. In the next section we will show how nonrelativistic effective field theories provide the solution.

2 Non Relativistic Effective Field Theories for Heavy Quarkonium

The reason for which the EFT approach is so successful for heavy quarkonium is the fact that heavy quarkonium, being a non-relativistic bound state, is characterized by a hierarchy of energy scales m , mv and mv^2 , with m the heavy-quark mass and $v \ll 1$ the relative heavy-quark velocity. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to the energy scales not relevant for the quarkonium system. Such integration is made in a matching procedure that enforces the complete equivalence between QCD and the EFT at a given order of the expansion in v and α_s .

Integrating out degrees of freedom of energy m , which

for heavy quarks can be done perturbatively, leads to non-relativistic QCD (NRQCD)[13, 14]. This EFT still contains the lower energy scales as dynamical degrees of freedom. In the last years, the problem of integrating out the remaining dynamical scales of NRQCD has been addressed by several groups and has reached a good level of understanding (a list of references can be found in [18]). The EFT obtained by subsequent matchings from QCD, where only the lightest degrees of freedom of energy mv^2 are left dynamical, is potential NRQCD, pNRQCD [16, 17]. This EFT is close to a quantum-mechanical description of the bound system and, therefore, as simple. It has been systematically explored in the dynamical regime $\Lambda_{\text{QCD}} \lesssim mv^2$ in [17, 19, 20] and in the regime $mv^2 \ll \Lambda_{\text{QCD}} \lesssim mv$ in [17, 21, 22]. The quantity Λ_{QCD} stands for the generic scale of non-perturbative physics.

Inside the EFT, the power counting in the small quantity v enables to select the operators that contribute to physical quantities up to a definite order in v . The EFT approach makes it possible, in the case of several observables, to achieve a rigorous factorization between the high-energy dynamics encoded into matching coefficients calculable in perturbation theory and the non-perturbative QCD dynamics encoded into few well-defined nonperturbative contributions to be fitted on the data or calculated on the lattice. Thus, several model independent QCD predictions become possible. I will detail these in the following section for NRQCD and pNRQCD.

3 Non Relativistic QCD

NRQCD is the EFT obtained by integrating out the hard scale m . The mass m being larger than the scale of non-perturbative physics, Λ_{QCD} , the matching to NRQCD can be done order by order in α_s . Hence, the NRQCD Lagrangian can be written as a sum of terms like $f_n O_n^{(d_n)}/m^{d_n-4}$, ordered in powers of α_s and v . More specifically, the Wilson coefficients f_n are series in α_s and encode the ultraviolet physics that has been integrated out from QCD. The operators $O_n^{(d_n)}$ of dimension d_n describe the low-energy dynamics and are counted in powers of v . Heavy quarkonium annihilations are controlled by the imaginary part of the NRQCD Hamiltonian, i.e. the imaginary part of the Wilson coefficients of the 4-fermion operators ($O_n^{(d_n)} = \psi^\dagger K_n \chi \chi^\dagger K'_n \psi$) in the NRQCD Lagrangian. The wave function of the quarkonium state is given by a series of terms in which the leading one is the quark antiquark in a color singlet state and the first correction, suppressed in v , comes from quark-antiquark in an octet state with a gluon:

$$|H\rangle = (|Q\bar{Q}_1\rangle + |Q\bar{Q}_8g\rangle + \dots) \otimes |nls\rangle. \quad (3)$$

To calculate physical quantities like spectra and decays the operators have to be evaluated over the wave functions and

the power counting of the two combines to give the order in v of the calculation. It is then clear that in the case in which the octet operators are enhanced with respect to the singlet, the octet part can be as relevant as the singlet one. The EFT contains naturally octet contributions.

3.1 Spectrum

The NRQCD Lagrangian is well suited for lattice evaluation. The quark propagators are the nonrelativistic ones and since we have integrated out the scale of the mass, the lattice step used in the simulation may be a factor $1/v$ bigger. Lattice evaluation of heavy systems like bottomonium become thus feasible. The latest results for the spectra (quenched and unquenched) are given e.g. in [15]. The radial splittings are accurate up to order $O(\alpha_s v^2)$ while fine and hyperfine splittings are accurate only up to $O(\alpha_s)$, due to the fact that only tree level matching coefficients have been used. A calculation of the NRQCD matching coefficients in the lattice regularization scheme is still missing and would be relevant to improve the precision of the lattice data.

3.2 Decays

NRQCD gives a factorization formula for heavy quarkonium (H) inclusive decay widths into light hadrons (LH) [14]

$$\Gamma(H \rightarrow LH) = \sum_n \frac{2 \text{Im} f_n}{m^{d_n-4}} \langle H | \psi^\dagger K_n \chi \chi^\dagger K'_n \psi | H \rangle. \quad (4)$$

Similar formulas hold for the electromagnetic decays. The 4-fermion operators are classified with respect to their rotational and spin symmetry (e.g. $O^{(2S+1)S_J}$, $O^{(2S+1)P_J}$, ...) and of their colour content (octet, O_8 , and singlet, O_1 , operators). Singlet operator expectation values may be easily related to the square of the quarkonium wave functions (or derivatives of it) at the origin. Octet contributions remain as nonperturbative matrix elements of operators over the quarkonium wave functions. According to the power counting of NRQCD, the octet contribution $\langle h | O_8(^1S_0) | h \rangle$ to P -wave decays is as relevant as the singlet contribution [14]. This octet contribution reabsorbs the dependence on the infrared cut-off of the Wilson coefficients solving the problem that arised in the color singlet potential model. Systematic improvements are possible, either by calculating higher-order corrections in the coupling constant or by adding higher-order operators. If one goes on in the expansion in v , that seems to be necessary for charmonium, the numbers of the involved nonperturbative matrix elements of octet operators over quarkonium states increases in such a way that limits the prediction power.

Besides this, precise theoretical predictions are also hampered by uncertainties in the NRQCD matrix elements and large corrections in NLO in α_s . The convergence of the perturbative series of the four-fermion matching coefficients is

indeed often bad (for examples see [24]). A solution may be provided by the resummation of the large contributions in the perturbative series coming from bubble-chain diagrams. This analysis has been successfully carried out in some specific cases in [25].

3.3 Production

As we have explained in the previous section, colour singlet production and colour singlet fragmentation underestimates the data on prompt quarkonium production at Fermilab by about an order of magnitude indicating that additional fragmentation contributions were missing [12]. This missing contribution is precisely the gluon fragmentation into colour-octet 3S_1 charm quark pairs. The probability to form a J/ψ particle from a pointlike $c\bar{c}$ pair in a colour octet 3S_1 state is given by a NRQCD nonperturbative matrix element which is suppressed by v^4 relative to the leading singlet term but is enhanced by two powers of α_s in the short distance part for producing color-octet quark pairs. When one introduces the leading colour-octet contributions, then the data of CDF can be reproduced. Still remains a puzzle the behaviour of the polarization at high p_t [12].

4 Potential Non Relativistic QCD

In NRQCD the dominant role of the potential as well as the quantum mechanical nature of the problem are not yet maximally exploited. A higher degree of simplification may be achieved building another effective theory for the low energy region of the non-relativistic bound-state, i.e. an EFT where only the ultrasoft degrees of freedom remain dynamical, while the rest is integrated out. We integrate out the scale of the momentum transfer $\sim mv$ which is supposed to be the next relevant scale. Then, two different situations may exist. In the first one, $mv \gg \Lambda_{\text{QCD}}$ and the matching from NRQCD to pNRQCD may be performed in perturbation theory, expanding in α_s . In the second situation, $mv \lesssim \Lambda_{\text{QCD}}$, the matching has to be nonperturbative, i.e. no expansion in α_s is allowed. Recalling that $r^{-1} \sim mv$, these two situations correspond to systems with inverse typical radius smaller or bigger than Λ_{QCD} , or systems respectively dominated by the short range or long range (with respect to the confinement radius) physics.

4.1 $mv \gg \Lambda_{\text{QCD}}$

The effective degrees of freedom are: $Q\bar{Q}$ states (that can be decomposed into a singlet and an octet wave function under color transformations) with energy of order of the next relevant scale, $\Lambda_{\text{QCD}}, mv^2$ and momentum \mathbf{p} of order mv , plus ultrasoft gluons $A_\mu(\mathbf{R}, t)$ with energy and momentum of order $\Lambda_{\text{QCD}}, mv^2$. All the gluon fields are multipole expanded (i.e. expanded in r). The Lagrangian is then an expansion in the small quantities p/m , $1/rm$ and $O(\Lambda_{\text{QCD}}, mv^2) \times r$.

The pNRQCD Lagrangian is given at the next to leading order in the multipole expansion by [17]:

$$\begin{aligned}
L_{\text{pNRQCD}} = & \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s(r) - \sum_{n=1} \frac{V_s^{(n)}}{m^n} \right) S \right. \\
& + O^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o(r) - \sum_{n=1} \frac{V_o^{(n)}}{m^n} \right) O \left. \right\} \\
& + gV_A(r) \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot \mathbf{E} S + S^\dagger \mathbf{r} \cdot \mathbf{E} O \right\} \\
& + g \frac{V_B(r)}{2} \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger O \mathbf{r} \cdot \mathbf{E} \right\} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}.
\end{aligned} \quad (5)$$

At the leading order in the multipole expansion, the singlet sector of the Lagrangian gives rise to equations of motion of the Schrödinger type. The two last lines of (5) contain (apart from the Yang-Mills Lagrangian) retardation (or non-potential) effects that start at the NLO in the multipole expansion. At this order the non-potential effects come from the singlet-octet and octet-octet interactions mediated by an ultrasoft chromoelectric field.

Recalling that $r \sim 1/mv$ and that the operators count like the next relevant scale, $O(mv^2, \Lambda_{\text{QCD}})$, to the power of the dimension, it follows that each term in the pNRQCD Lagrangian has a definite power counting. From the power counting e.g., it follows that the interaction of quarks with ultrasoft gluons is suppressed in the Lagrangian by v (by gv if $mv^2 \gg \Lambda_{\text{QCD}}$) with respect to the LO.

The singlet and octet potentials are well defined objects to be calculated in the perturbative matching. In this way a determination of the singlet $Q\bar{Q}$ potential at three loops leading log has been obtained in [19] and consequently also a determination of α_V which shows how this quantity starts to depend on the infrared behaviour of the theory at three loops.

4.1.1 Spectrum

Given the Lagrangian in (5) it is possible to calculate the quarkonium energy levels at order $m\alpha_s^5$ [19, 20]. At this order the energy E_n of the level n receives both from the average value of the potentials and from the singlet-octet interaction (retardation effect) a contribution that read

$$E_n = \langle n | V_s(\mu) | n \rangle - ig^2 \int_0^\infty dt \langle n | \mathbf{r} e^{it(H_s - H_o)} \mathbf{r} | n \rangle \langle \mathbf{E}(t) \mathbf{E}(0) \rangle (\mu) (6)$$

being H_S and H_o the singlet and octet Hamiltonian respectively. The nonlocal electric correlator is a nonperturbative object dominated by the scale Λ_{QCD} , $\langle \mathbf{E}(t) \mathbf{E}(0) \rangle (\mu) \sim \exp\{-\Lambda_{\text{QCD}} t\}$. Thus the integral in (6) is the convolution of two exponentials with exponent of order mv^2 (for the energy) or Λ_{QCD} (for the correlator). Depending on the relative relation of the two scales three different situations take place:

- if $mv^2 \gg \Lambda_{\text{QCD}}$ then the correlator reduces to the local gluon correlator and the second contribution in (6) corresponds to the previously mentioned Voloshin-Leutwyler sum rule contribution [11].
- if $mv^2 \ll \Lambda_{\text{QCD}}$ then the energy exponential can be expanded and the second contribution in (6) corresponds to a short range nonperturbative potential corrections [17].
- if $mv^2 \sim \Lambda_{\text{QCD}}$ then neither exponentials can be expanded and the nonlocal condensate has to be used in the energy level calculation [17].

Both the potential model and the sum rule results are contained in pNRQCD as different kinematical limits. What appeared as a puzzle and a problem at the origin is now understood as a consequence of the richness of the quarkonium dynamics and is appropriately accounted for by the EFT.

The calculation of the quarkonium energy levels at higher orders in perturbation theory is relevant to extract the masses of the heavy quarks from the $\Upsilon(1S)$, J/ψ and $t\bar{t}$ production cross section cf. [26, 20]. The perturbative determination of the levels, have been used in [27, 28] for the calculation of the energy levels of some lowest resonances of bottomonium, charmonium and B_c , after having removed the renormalon (between the pole mass and the singlet static potential) and under the assumption (or to test the assumption) that the nonperturbative corrections, in the form of nonlocal condensates or short range nonperturbative potentials, are in total small.

4.2 $\Lambda_{\text{QCD}} \sim mv$

In this case the (nonperturbative) matching to pNRQCD has to be done in one single step [21]. Under the circumstances that other degrees of freedom (like those associated with heavy-light meson pair threshold production and heavy hybrids) develop a mass gap of order Λ_{QCD} the quarkonium singlet field S remains as the only dynamical degree of freedom in the pNRQCD Lagrangian, which reads [17, 21, 22]

$$L_{\text{pNRQCD}} = \text{Tr} \left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_S(r) \right) S \right\} \quad (7)$$

In this regime we recover the quark potential singlet model from pNRQCD [21, 22]. The final result for the potentials (static and relativistic corrections) appears factorized in a part containing the high energy dynamics (and calculable in perturbation theory) which is inherited from the NRQCD matching coefficients, and a part containing the low energy dynamics given in terms of Wilson loops and chromo-electric and chromo-magnetic insertions in the Wilson loop [21]. Such low energy contributions can be

calculated on the lattice [30] or evaluated in QCD vacuum models [7, 31]. The expression obtained for the potential is the QCD expression, in particular all the perturbative contributions to the potential at the hard scale are correctly taken into account. This solves the problem of consistency with perturbative one-loop calculations that was previously encountered in the Wilson loop approach. Moreover, further contributions, including a $1/m$ nonperturbative potential, appear with respect to the Wilson loop original results [6].

4.2.1 Decays

The inclusive quarkonium decay width achieve in pNRQCD a factorization with respect to the wave function (or its derivatives) calculated in zero which is suggestive of the early potential models results: (cf. eq. (2))

$$\Gamma(H \rightarrow LH) = F(\alpha_s, \Lambda_{\text{QCD}}) \cdot |\psi(0)|^2. \quad (8)$$

Similar expressions hold for the electromagnetic decays. However, the coefficient F depends here both on α_s and Λ_{QCD} . In particular all NRQCD matrix elements, including the octet ones, can be expressed through pNRQCD as products of universal nonperturbative factors by the squares of the quarkonium wave functions (or derivatives of it) at the origin. The nonperturbative factors are typically integral of nonlocal electric or magnetic correlators and thus depending on the glue but not on the quarkonium state [22]. The presence of this nonperturbative correlators is indicated by the Λ_{QCD} dependence of F in (8). Typically F contains both the NRQCD matching coefficients f at the hard scale m and the nonperturbative correlators at the low energy scale Λ_{QCD} . The nonperturbative correlators, being state independent, are in a smaller number than the nonperturbative NRQCD matrix elements and thus the predictive power is greatly increased in going from NRQCD to pNRQCD. In [22] the inclusive decay widths into light hadrons, photons and lepton pairs of all S -wave and P -wave states (under threshold) have been calculated up to $O(mv^3 \times (\Lambda_{\text{QCD}}^2/m^2, E/m))$ and $O(mv^5)$. A large reduction in the number of unknown nonperturbative parameters is achieved and, therefore, after having fixed the nonperturbative parameters on charmonium decays, new model-independent QCD predictions are given for the bottomonium decay widths [22].

Once the methodology to compute the potentials (real and imaginary contributions) and from these the inclusive decays, within a $1/m$ expansion in the matching has been developed, the next question to be addressed is to which extent one can compute the *full* potential within a $1/m$ expansion in the case $\Lambda_{\text{QCD}} \gg mv^2$. It has been shown [23] that new non-analytic terms arise due to the three-momentum scale $\sqrt{m\Lambda_{\text{QCD}}}$. These terms can be incorporated into local potentials ($\delta^3(\mathbf{r})$ and derivatives of it) and scale as half-integer powers of $1/m$. Moreover, it is possible to factorize these effects in a model independent way and compute

them within a systematic expansion in some small parameters [23].

4.2.2 Production

Since the power counting of pNRQCD may be different from the power counting of NRQCD, we expect that we may eventually explain in this way some of the difficulties that NRQCD is facing in explaining the polarization of the prompt J/ψ data. In particular, if the magnetic field turns out to be not suppressed with respect to the electric field operator in the power counting, then the spin flip term is enhanced and the polarization may be diluted explaining the behaviour of the data with high p_t [22, 21, 29].

4.3 Renormalization group improvement and Poincaré invariance

The effective field theory can be used for a very efficient resummation of the large logs (typically logs of the ratio of energy and momentum scales) once a renormalization groups analysis of the EFT has been performed. Such program has been successfully performed in pNRQCD [32].

Since the EFTs are constructed to be equivalent to QCD, Poincaré invariance has still to hold. In [33] the constraints induced by the algebra of the Poincaré generators on non-relativistic effective field theories have been discussed. It has been shown that Poincaré invariance imposes well defined relations among the EFT matching coefficients. The relations have been given both for NRQCD and for pNRQCD.

5 Conclusions

The progress in our understanding of non-relativistic effective field theories makes it possible to move beyond *ad hoc* phenomenological models and have a unified description of the different heavy-quarkonium observables, so that the same quantities determined from a set of data may be used in order to describe other sets. The old puzzles and problems have been clarified and have been understood inside the EFT formulation. Predictions based on non-relativistic EFTs are conceptually solid, and systematically improvable. EFTs put quarkonium on the solid ground of QCD: quarkonium becomes a privileged window for precision measurements, new physics, confinement mechanism investigations [1].

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